AN ADAPTIVE CONTROL SCHEME FOR A FLEXIBLE MANIPULATOR

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Abstract

The problem of controlling a single link flexible manipulator is considered. A self-tuning adaptive control scheme is proposed which consists of a least squares on-line parameter identification of an equivalent linear model followed by a tuning of the gains of a pole placement controller using the parameter estimates. Since the initial parameter values for this model are assumed unknown, the use of arbitrarily chosen initial parameter estimates in the adaptive controller would result in undesirable transient effects. Hence, the initial stage control is carried out with a PID controller. Once the identified parameters have converged, control is transferred to the adaptive controller. Naturally, the relevant issues in this scheme are tests for parameter convergence and minimization of overshoots during control switch-over. To demonstrate the effectiveness of the proposed scheme, simulation results are presented with an analytical nonlinear dynamic model of a single link flexible manipulator.

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1 INTRODUCTION

Automated manipulation is finding increasing use in production, military and space industries for performing routine, monotonous and hazardous tasks. The present day manipulators can perform with sufficiently adequate accuracy at the expense, however, of payload capacity and operating speed. One possible cost-effective solution is to build manipulators with lighter links. The lightweight links reduce the moment of inertia at each joint, permit the use of direct drive motors and have the advantages of manufacturing simplicity and lower cost. The next generation of manipulators would naturally have to be flexible. Mechanical flexiblity, however, generates a fairly severe problem of control of the manipulator end effector motion in its work space. This is due to the inevitable excitation of structural vibrations and the resulting interactions between these vibrations and the control action which would effect the accuracy required of the manipulator.

The successful implementation of flexible robots is contingent upon achieving acceptably uniform performance with regard to variations in load , task specification, reasonable speeds and the ability to compensate for any environmental disturbances. In contrast to robots consisting of rigid links, the dynamic behavior of flexible manipulators is not easy to characterize, especially under conditions of high speed and large amplitude motion. It is not only the fact that this behavior is described by highly nonlinear differential equations but also the lack of a precise knowledge of this description that makes the design of an acceptable control system, over the total range of its operation, a formidable task. The dynamic effects due to changes in configuration, load and moments of inertia, higher speed and unpredictable disturbances tend to degrade the performance of the flexible manipulator arm. The control scheme that is to be developed, therefore, has to overcome these dynamic effects.

This paper attempts at a resolution of difficulties posed by this problem by employing a self-tuning control approach. The strategy here briefly consists of (i) a least squares on-line parameter identification of an equivalent linear model, followed by (ii) a tuning of the controller gains by an adaptive control algorithm throughout

the range of the manipulator operation. Thus any changes occuring in the manipulator's dynamic description will automatically be reflected in the parameter estimates and would, therefore, be counteracted by updating the controller gains.

An important step prior to parameter identification is to obtain a valid model structure of the manipulator dynamics. This is derived by analytical modelling based on Lagrange's equation and assumed mode shape functions from the finite element method. This nonlinear analytical model is used to generate the input/output data which, in turn, is employed in the least squares parameter estimation. Since, initially, the parameters are assumed to be unknown, the parameter estimates obtained during this initial stage would be unsuitable for updating the controller gains. Hence, during this initial stage, a simple PID stabilizing controller is used with the manipulator model and the parameter identification process is initiated. On convergence of the parameters, the control action is switched over to the adaptive controller. A salient feature of the present work involves the implementation of a convergence test to minimize any undesirable transient effects following the switch-over.

2 THE ADAPTIVE POLE PLACEMENT CONTROLLER

The control scheme considered here is based on adaptive pole placement. While a variety of configutations can be found in the literature ([1], [3], [4]) for pole placement, the one involving a Luenberger observer structure (Fig. 1) as suggested by Elliot and Wolovich [1] is used here. This choice is based on the fact that it results in a closed loop system of the same order as the open-loop system (due to pole-zero cancellations). Also it does not add any undesirable zeros to the plant as might happen with the structure suggested in [3].

The adaptive pole-placement concept is briefly presented below in a discrete-time framework:

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Let the plant to be controlled have the transfer function

$$\frac{B(q^{-1})}{A(q^{-1})}. \qquad (1)$$

where q^{-1} is the backward shift operator, and

$$A(q^{-1}) = 1 + \sum_{i=1}^{n} a_i q^{-i} \qquad (2)$$

$$B(q^{-1}) = \sum_{i=1}^{n} b_i q^{-i} \qquad (3)$$

so that it has the description

$$A(q^{-1})y(t) = B(q^{-1})u(t). \qquad (4)$$

where u(t) and y(t) are the input and output respectively.

The Adaptive Pole Placement Algorithm :

From the structure of Figure 1, one can formulate the following equations.

$$Q(q^{-1})g(t) = \hat{K}(t, q^{-1})u(t) + \hat{H}(t, q^{-1})y(t) \qquad (5)$$

$$u(t) = g(t) + v(t) \qquad (6)$$

where

$$\hat{K}(t, q^{-1}) = \sum_{i=1}^{n} \hat{k}_i(t)q^{-i} \qquad (7)$$

$$\hat{H}(t, q^{-1}) = \sum_{i=1}^{n} \hat{h}_i(t)q^{-i} \qquad (8)$$

and

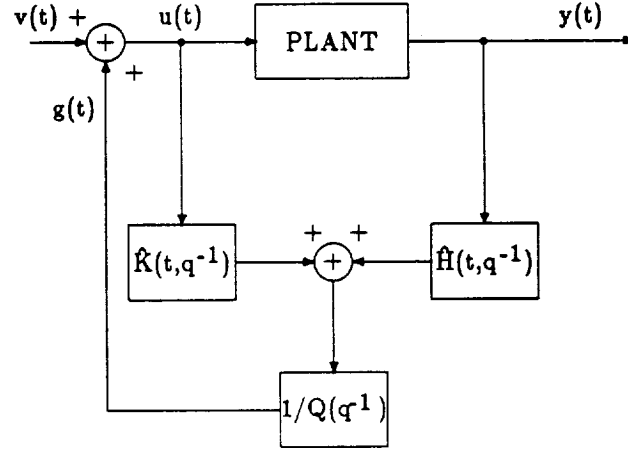$$Q(q^{-1}) = 1 + \sum_{i=1}^{n} q_i q^{-i} \qquad (9)$$

Figure 1: Luenberger observer structure for pole placement control

Let

$$\hat{A}(t, q^{-1}) = 1 + \sum_{i=1}^{n} \hat{a}_i(t) q^{-i} \qquad (10)$$

$$\hat{B}(t, q^{-1}) = \sum_{i=1}^{n} \hat{b}_i(t) q^{-i} \qquad (11)$$

where $\hat{a}_i(t)$ and $\hat{b}_i(t)$ are the estimates of a_i and b_i.

If $\hat{K}(t, q^{-1})$ and $\hat{H}(t, q^{-1})$ are made to satisfy the following relation

$$
\begin{aligned}
& \hat{H}(t, q^{-1}) \hat{B}(t, q^{-1}) + \hat{K}(t, q^{-1}) \hat{A}(t, q^{-1}) \\
= \ & Q(q^{-1})[\hat{A}(t, q^{-1}) - A_d(t, q^{-1})]
\end{aligned} \qquad (12)
$$

then the resulting closed-loop transfer function becomes

$$\frac{B(q^{-1})}{A_d(t, q^{-1})} \qquad (13)$$

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when the identified parameters converge to the plant parameters, where

$$A_d(t, q^{-1}) = 1 + \sum_{i=1}^{n} A_{di}(t)q^{-i} \tag{14}$$

With this structure, however, the plant cannot be made to track a step input signal. In order to equip this structure with such a tracking facility, unity feedback is applied and an integrator is inserted in the forward path. This can be formulated as

$$w(t) = s(t) \cdot c + w(t-1) \tag{15}$$

$$s(t) = v(t) - y(t) \tag{16}$$

Also equation (6) should be modified to :

$$u(t) = g(t) + w(t) \tag{17}$$

The desired denominator $A_d(t, q^{-1})$ and the scalar gain c can be determined from the desired closed-loop denominator $D(q^{-1})$ by the following equations.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & \hat{b}_1 \\ -1 & 1 & 0 & 0 & \cdots & \hat{b}_2 \\ 0 & -1 & 1 & 0 & \cdots & \hat{b}_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -1 & 1 & \hat{b}_n \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} A_{d1} \\ A_{d2} \\ A_{d3} \\ \vdots \\ A_{dn} \\ c \end{pmatrix} = \begin{pmatrix} d_1 + 1 \\ d_2 \\ d_3 \\ \vdots \\ d_n \\ d_{n+1} \end{pmatrix} \tag{18}$$

where

$$D(q^{-1}) = 1 + \sum_{i=1}^{n+1} d_i q^{-i} \tag{19}$$

Since we can obtain $A_d(t, q^{-1})$ from equation (18) , the $\hat{H}(t, q^{-1})$ and $\hat{K}(t, q^{-1})$ can be obtained from equation (12).

The block diagram of this scheme is shown in figure 2.

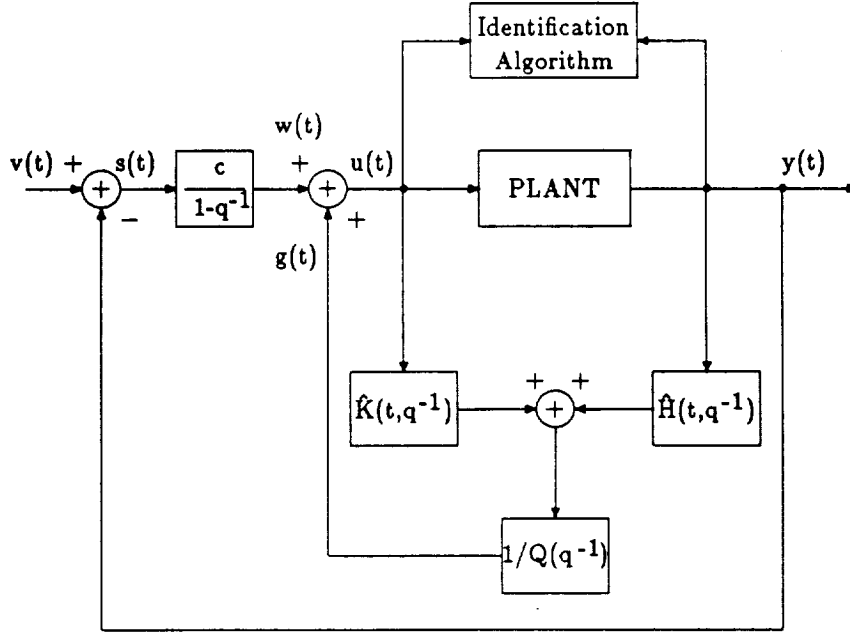Note that the step input tracking facility is achieved by increasing the order of the overall system to only n+1.

Figure 2: Adaptive Pole Placement Control Scheme Using Luenberger Observer Structure with Integral Action

The Least Squares Identification Algorithm:

The estimates $\hat{a}_i$ and $\hat{b}_i$ used in the control scheme are obtained by a least squares parameter identification algorithm [4] as follows:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-2)\Phi(t-1)[y(t) - \Phi(t-1)^T\hat{\theta}(t-1)]}{1 + \Phi(t-1)^T P(t-2))\Phi(t-1)}; \quad (20)$$

$$t \geq 1$$

$$P(t-1) = P(t-2) - \frac{P(t-2)\Phi(t-1)\Phi(t-1)^T P(t-2)}{1 + \Phi(t-1)^T P(t-2)\Phi(t-1)} \quad (21)$$

with $\hat{\theta}(0)$ given and $P(-1)$ is any positive definite matrix, where

$$\hat{\theta}(t) = [-\hat{a}_1(t), -\hat{a}_2(t), \cdots, -\hat{a}_n(t), \hat{b}_1(t), \hat{b}_2(t), \cdots, \hat{b}_n(t)]^T$$

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is the current parameter estimate vector, and

$$\Phi(t-1) = [y(t-1), y(t-2), \cdots, y(t-n), u(t-1), u(t-2), \cdots, u(t-n)]^T$$

3 SWITCH-OVER FROM PID TO ADAPTIVE CONTROLLER

A critical question in the present control scheme is to determine an appropriate time to switch from the initial PID controller to the adaptive controller. The simplest way is to wait till the parameter estimates resulting from the identification algorithm have converged to their true values. The following criterion provides a check on such a convergence.

The Convergence Criterion:

Assume $\| \tilde{\theta}(0) \|^2 \leq M$.
If

$$\lambda_{max}[P(t-1)] \leq \frac{\epsilon \lambda_{min}[P(-1)]}{M} \qquad (22)$$

then

$$\| \tilde{\theta}(t) \|^2 \leq \epsilon, \qquad (23)$$

where

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta$$

$\hat{\theta}(t)$: identified parameter vector at time t
θ : actual parameter vector =
$\qquad [-a_1, -a_2 \cdots, -a_n, b_1, b_2, \cdots, b_n]^T$
ϵ : the error tolerence for the convergence test
$\qquad$ of the identified parameters.
$\lambda_{max}[P(t-1)]$: maximum eigen value of $P(t-1)$.

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Proof:

From [4] (p. 61), one can get the following inequality

$$\lambda_{min}[P(t-1)^{-1}]\| \tilde{\theta}(t) \|^2 \leq \lambda_{max}[P(-1)^{-1}]\| \tilde{\theta}(0) \|^2 \qquad (24)$$

which implies

$$\| \tilde{\theta}(t) \|^2 \leq \frac{\lambda_{max}[P(t-1)]}{\lambda_{min}[P(-1)]}\| \tilde{\theta}(0) \|^2 \qquad (25)$$

Using (22) and (25), it follows that

$$\| \tilde{\theta}(t) \|^2 \leq \frac{\epsilon\| \tilde{\theta}(0) \|^2}{M} \leq \epsilon \qquad (26)$$

Thus by computing $\lambda_{max}[P(t-1)]$, one can test the convergence of the parameter estimates.

Switching Logic:

Once the identified parameters have converged to their true values and the system step response has reached steady state, control action is switched over from the PID to the adaptive controller. This is probably the simplest manner to implement the switchover without causing any undesirable transients.

An alternative switching logic is proposed here which does not require the step response to reach steady state. However, this logic is limited only to those systems that satisfy the conditions for one-step-ahead control [4].

Assume

(i) the plant to be linear time invariant,

(ii) the switching instant to correspond to $t = 0$, and

(iii) the desired output trajectory after switching to be the same as the one that would have been obtained, had the adaptive controller been applied to the plant starting at rest from that position(y_o), where

$$y_o = [y(-1), y(-2), \cdots, y(-n), u(-1), u(-2), \cdots, u(-n)]^T \theta \qquad (27)$$

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is the output at the switching instant.

In order to satisfy the last assumption, a correction input $u_c(t)$ is needed to compensate for the terminal conditions resulting from the PID controller. Thus the plant input would be

$$u(t) = u_d(t) + u_c(t) \qquad (28)$$

where $u_d(t)$ is the input generated by the pole placement algorithm.

The plant output can be expressed as

$$y(t) = y_d(t) + y_c(t) \qquad (29)$$

where

$$y_d(t) = \Phi_d^T(t)\theta \quad ; y_d(0) = 0 \qquad (30)$$

$$y_c(t) = \Phi_c^T(t)\theta \quad ; y_c(0) = y_o \qquad (31)$$

where the subscripts denote the correspondence of the two components.

From assumption (iii), $y_c(t) = y_o$ for $t \geq 0$, and the compensating input $u_c(t)$ is obtained using (31) as:

$$u_c(t) = \frac{1}{b_1}\{y_o - [y_c(t), y_c(t-1), \cdots, y_c(t-n+1), 0, u_c(t-1), \cdots, u_c(t-n+1)]^T\theta\}$$
$$(32)$$

With a proper choice of the sampling interval, the flexible manipulator discrete model is found to meet the requirements of one-step-ahead control. However, this approach is found to be suitable only in those situations where the deflections are small, and is not used in the simulation here.

The complete control block diagram is shown in Fig. 3 .

4   SIMULATION RESULTS

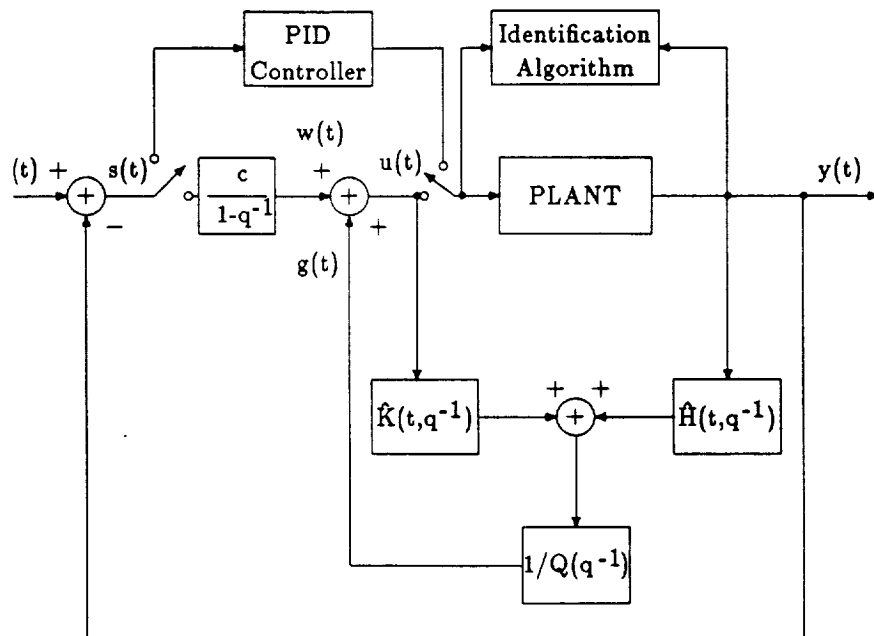The dynamic analytical model of the single link flexible manipulator is described by [5]:

ure 3: Complete Block Diagram of PID-Adaptive Pole Placement Controller

$$\begin{pmatrix} (J + \sum_{i=1}^{N} m_{ii} r_i^2) & m_{1\beta} & m_{2\beta} & \cdots & m_{N\beta} \\ m_{1\beta} & m_{11} & 0 & \cdots & 0 \\ m_{2\beta} & 0 & m_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ m_{N\beta} & 0 & 0 & \cdots & m_{NN} \end{pmatrix} \begin{pmatrix} \ddot{\beta} \\ \ddot{r}_1 \\ \ddot{r}_2 \\ \vdots \\ \ddot{r}_N \end{pmatrix} = \begin{pmatrix} u - 2\sum_{i=1}^{N} m_{ii} r_i \dot{r}_i \\ (\dot{\beta}^2 - \omega_1^2) r_1 \\ (\dot{\beta}^2 - \omega_2^2) r_2 \\ \vdots \\ (\dot{\beta}^2 - \omega_N^2) r_N \end{pmatrix}$$

(33)

$$y = tan^{-1} \frac{w(l,t)}{l} + \beta \qquad (34)$$

$$w(x,t) = \sum_{i=1}^{N} \phi_i(x) r_i(t) \qquad (35)$$

where

u : input torque to the beam
y : tip position
l : the length of the beam
β : the hub angle
$\phi_i(x)$: the mode shape functions of the beam
$r_i(t)$: the generalized coordinates

The desired closed-loop denominator for the adaptive pole placement controller is chosen as

$$D(q^{-1}) = 1 + d_1 q^{-1} + d_2 q^{-2} \qquad (36)$$

where

$$d_1 = -2e^{-\xi \omega T} cos \, \omega T \sqrt{1 - \xi^2} \qquad (37)$$

$$d_2 = e^{-2\xi \omega T} \qquad (38)$$

where T is the sampling period in seconds.

For computer simulation, the following numerical values are used: $n = 4$, $N = 2$, $\omega = 5$, $\xi = 1$, $T = 0.1$, $P(-1) = 10^8 I_{8\times 8}$, $M = 10$ and $\epsilon = 0.7$. The switching from PID to the adaptive controller occurs at $t = 4$ secs.
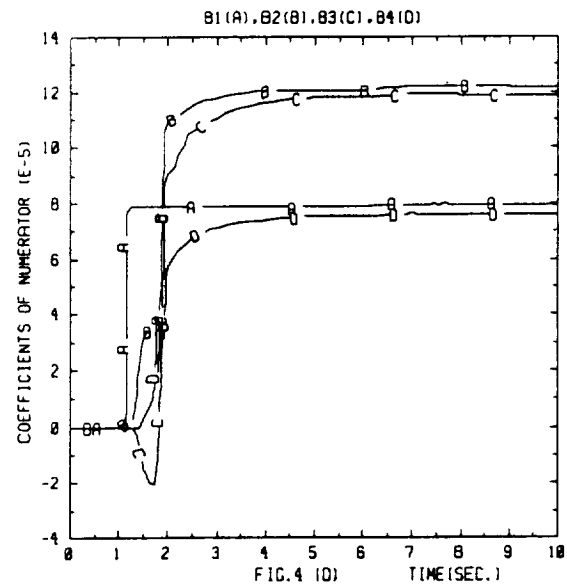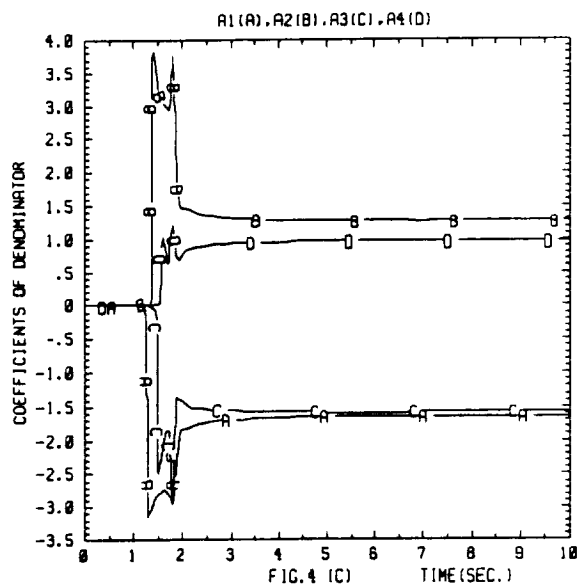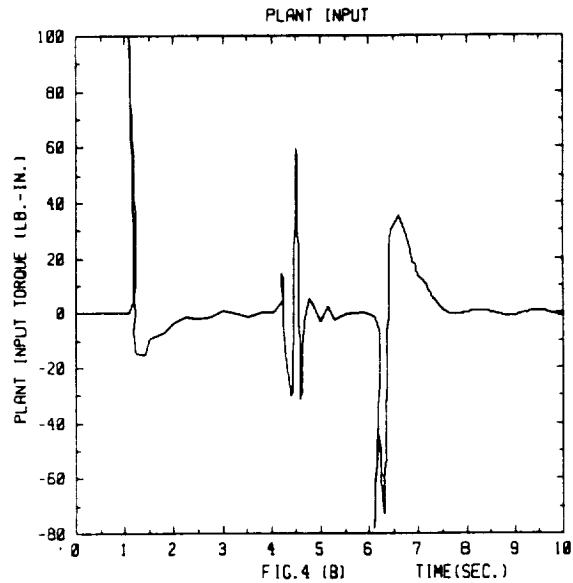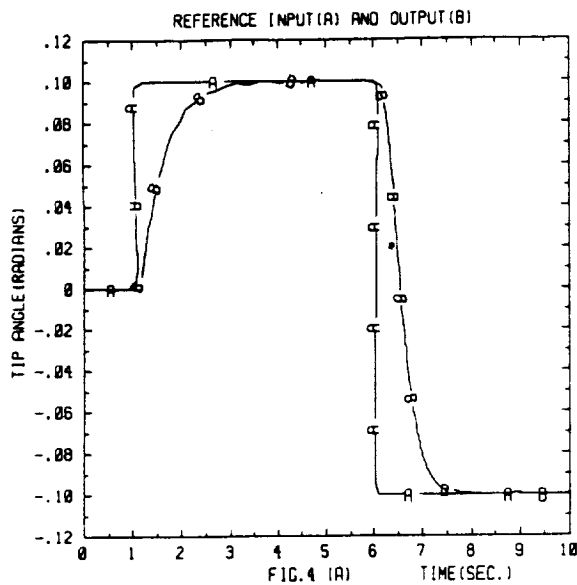
The results are shown in Fig. 4.

Figure 4: Simulation of Combined PID and Adaptive Pole Placement Control:
(A) reference input v(t) and plant output y(t),
(B) plant input u(t),
(C) convergence of identified coefficients of denominator,
(D) convergence of identified coefficients of numerator.

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5 CONCLUSION

A simulation based study for the adaptive control of a single link flexible manipulator has been considered. Such a control approach is of practical importance since the dynamic characteristics of the manipulator change considerably especially while picking up or releasing payloads. In such cases unless the control gains are suitably updated, the performance would be poor.

Since the adaptive control scheme depends on the parameter estimates from an on-line identification algorithm, the initial control action is carried out with a PID controller during which the identification process is initiated. On convergence of the parameter estimates, control is smoothly transferred to the adaptive controller. A criterion for testing the convergence has been presented. The simulation results amply demonstrate the effectiveness of the proposed scheme.

Experimental verification of the control scheme on a laboratory test set-up is presently in progress.

References

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